

Simplified Treatment of Collective Instabilities in Matter ^{*}

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A formal analysis was made of the onset of instabilities in dilute nuclear matter in which collective modes are agitated by stochastic nucleon-nucleon collisions and then exponentially amplified by the self-consistent field. Moreover, employing approximate expressions for the transport coefficients [1], simple expressions were derived for the key quantities, so that survey calculations are facilitated.

It is convenient to discuss instabilities in uniform matter in terms of the Landau parameter $F_0 = (3\rho/2\epsilon_F)(\partial h/\partial\rho)$. When harmonic modes in matter are considered, this quantity generalizes to $F_0(k) \equiv (3\rho/2\epsilon_F)(\partial h_k/\partial\rho)$, involving the appropriate Fourier component of the self-consistent response. Further generalization is useful at finite T ,

$$F_T(k) \equiv \phi_0 \frac{\rho}{T} \frac{\partial h_k}{\partial \rho} \approx F_0(k) \left[1 - \frac{\pi^2}{12} \left(\frac{T}{\epsilon_F} \right)^2 \right], \quad (1)$$

employing the Sommerfeld expansion of the Fermi-surface moment ϕ_0 [2]. The dispersion relation for the collective growth time $t_k \equiv m/kP_F\gamma_k$ is then to a good approximation given by $1 = F_T(k)(\gamma_k \arctan(1/\gamma_k) - 1)$, which can readily be solved by iteration:

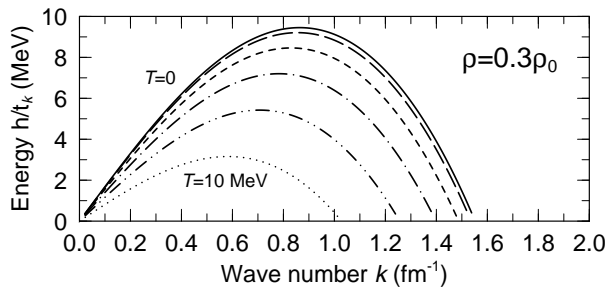


Figure 1: Characteristic energies for the collective modes in unstable matter, $E_k = \hbar/t_k$, with the generalized Seyler-Blanchard model [3].

The random collisions provide a source term generating density irregularities which are then amplified by the unstable mean field h [4], leading quickly from a linear growth to an explosive development, $\sigma_k \approx \mathcal{D}_k \exp(2t/t_k)$:

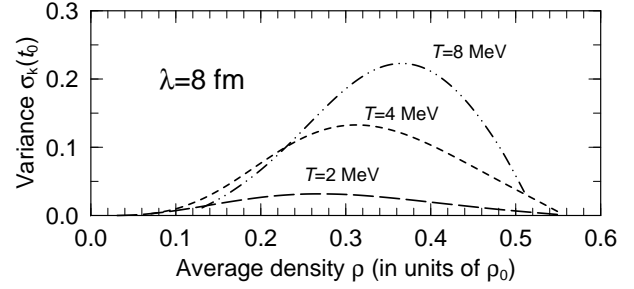


Figure 2: The magnitude of the variance σ_k of density fluctuations having the wave length $\lambda = 2\pi/k = 8$ fm (near which the most rapid growth occurs), after a given time $t_0 = 10^{-22}$ s, as a function of the average density ρ and for specified temperatures T .

The derived analytical approximations facilitate the analysis of the onset of fragmentation in the spinodal zone of the phase diagram. Both the source terms for the fluctuations and the amplification times can be easily obtained. Several additional results are also useful, including the expansion of the angular quantities on complex Legendre polynomials, which is helpful for understanding the multipolarity properties.

[1] J. Randrup and S. Ayik, Nucl. Phys. A572 (1994) 489

[2] J. Randrup, *Statistical Description of Transport in Plasma, Astro, and Nuclear Physics*, Les Houches, France (1992), Nova Science, p. 353

[3] E. de Lima Medeiros and J. Randrup, Phys. Rev. C45 (1991) 372

[4] M. Colonna, Ph. Chomaz, and J. Randrup, Nucl. Phys. A567 (1994) 637

^{*} LBL-35847: Nucl. Phys. A583 (1995) 329